

# The Solution of Multi-scale HVDC Geoelectric Current Field by Domain Decomposition Method Based on GMRES Iterative Algorithm

Ruixiang Tao, Zezhong Wang

Key Laboratory of High Voltage & EMC, Beijing, 102206, China, r.x.tao@ncepu.edu.cn

Along with the large-scale construction of HVDC project, the negative influence which are brought by DC unbalanced operation can't be ignored. In order to obtain a more accurate distribution of HVDC geoelectric current field, we should build a model that is closer to real entity. This paper presents a model that take the specific size of the ground electrode into account. Obviously, the ground electrode is much smaller in size than earth that current flows in. This constitutes a multi-scale problem. Here the domain decomposition method (DDM) is used to solve that multi-scale problem, a small-scale domain including the earth electrode is decomposed from the whole model, and the finite element method is used to solve each model. Generalized Minimal Residual (GMRES) is put forward to be the iterative algorithm for DDM, it is proved that GMRES is more efficient than direct iteration and relaxation iteration. Finally, we obtain a more accurate distribution of the current field of the HVDC grounding electrode.

*Index Terms*—Algorithm design and analysis, Electrical engineering computing , Electrodes, Finite element analysis the current injected into the earth from the ground electrode is taken as  $J_0$ .

## I. INTRODUCTION

HVDC is widely used for long-distance, large-capacity transmission of electricity. However when HVDC is under unipolar operation or in malfunction condition, negative influence which are brought by DC unbalanced operation can't be ignored. Distribution of DC earth surface potential will induce step voltage, which could affect human and animal safety. In order to obtain a more accurate distribution of HVDC geoelectric current field, we should build a model that is closer to real entity. Many methods can be taken into account to work out the distribution of HVDC geoelectric current field, such as FEM, Image Method, Green Function Method and so on. It is difficult for traditional FEM to take the specific size of the ground electrode into account in the model because of large amount of calculation and high computing complexity. Image Method and Green Function Method cannot deal with the earth model with complex resistivity distribution. This paper presents a model that take the specific size of the ground electrode into account. Obviously, the ground electrode is much smaller in size than earth model, those constitutes a multi-scale problem. Here the domain decomposition method (DDM) is taken to solve that multi-scale problem. As for the iteration of DDM, GMRES<sup>[1-2]</sup> is put forward to achieve the purpose of improving the iterative efficiency and feasibility.

## II. GENERAL DESCRIPTION OF DDM FOR THE MODEL

The domain decomposition used in this paper is the overlapping domain decomposition. For this model, domain that contains ground electrode and a small part of earth will be separated from the whole calculation model and the rest only contains earth that is large in size, as shown in Figure 1. The overlapping domain lies between the two cuboids with the dimension  $8\text{ km} \times 8\text{ km} \times 4\text{ km}$  and  $12\text{ km} \times 12\text{ km} \times 6\text{ km}$  separately. The potential of the surface of two cuboids except the top surface can be named as  $\varphi_1$  and  $\varphi_2$  separately. The potential of the boundary of whole model is taken as  $\varphi_0$  and

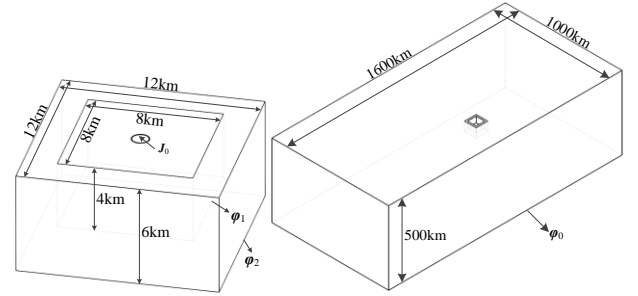


Fig. 1. Small scale model and large scale model

## III. GENERAL PRINCIPLE OF GMRES FOR THE ITERATION OF DDM FOR THE MODEL

The relationship among  $\varphi_1$ ,  $\varphi_2$ ,  $\varphi_0$  and  $J_0$  can be written in function form as follows.

$$\begin{cases} \varphi_1 = \mathbf{B}\varphi_2 + \mathbf{C}J_0 \\ \varphi_2 = \mathbf{D}\varphi_1 + \mathbf{E}\varphi_0 \end{cases} \quad (1)$$

Rewrite the relation as follows

$$\begin{cases} \varphi_1 - \mathbf{B}\varphi_2 = \mathbf{C}J_0 \\ \varphi_2 - \mathbf{D}\varphi_1 = \mathbf{E}\varphi_0 \end{cases} \quad (2)$$

Rewrite above functions as matrix form

$$\begin{pmatrix} \mathbf{I} & -\mathbf{B} \\ -\mathbf{D} & \mathbf{I} \end{pmatrix} \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} = \begin{pmatrix} \mathbf{C}J_0 \\ \mathbf{E}\varphi_0 \end{pmatrix} \quad (3)$$

Simplify that matrix form as linear equation group

$$\mathbf{A}\mathbf{x} = \mathbf{b} \quad (4)$$

HVDC geoelectric current field belongs to steady-state current field, so matrix  $\mathbf{A}$ ,  $\mathbf{x}$  and  $\mathbf{b}$  are all real matrices. Here,  $\mathbf{x}$  contains  $\varphi_1$  and  $\varphi_2$ . That is to say  $\varphi_1$  and  $\varphi_2$  are used as the iteration variable.

Then GMRES iterative algorithm is taken for solving equations without computing the specific coefficient matrix  $\mathbf{A}$  directly. The dimension of the vector  $\mathbf{x}$  is assumed to be  $n$ , iterative potential  $\varphi_1$ ,  $\varphi_2$  contain  $n_1$ ,  $n_2$  elements separately, and  $n_1 + n_2 = n$ .

- 1) Take the initial iteration vector  $\mathbf{x}^{(0)}=\mathbf{0}$ , i.e.  $\varphi_1^{(0)}=\mathbf{0}$ ,  $\varphi_2^{(0)}=\mathbf{0}$ . In small scale domain, solve the domain by means of FEM with the boundary condition  $\varphi_2^{(0)}=\mathbf{0}$  and current density  $\mathbf{J}_0$ , then generate  $\varphi_1^{(0)}$ . For  $\varphi_2^{(0)}=\mathbf{0}$ , then  $\mathbf{B}\varphi_2^{(0)}=\mathbf{0}$ , from (2),  $\varphi_1^{(0)}$  equals to  $\mathbf{C}\mathbf{J}_0$ . Similarly, in large scale domain, solve the domain with the boundary condition  $\varphi_1^{(0)}=\mathbf{0}$  and  $\varphi_0$ , then, generate  $\varphi_2^{(0)}$ , finally we get the initial residue  $\mathbf{r}_0$  that contains  $\varphi_1^{(0)}$  and  $\varphi_2^{(0)}$ . Normalize  $\mathbf{r}_0$ , we obtain  $\mathbf{v}^{(1)}$ .
- 2) According to correspondence, take the last  $n_2$  elements of  $\mathbf{v}^{(j)}$  ( $j=1,2,\dots,m$ ) to assign to  $\varphi_2^{(j)}$ , and set  $\mathbf{J}_0=\mathbf{0}$ . Solving the small scale domain by means of FEM, and generate  $\varphi_1^{(j)}$ , from (2),  $\varphi_1^{(j)}$  equals to  $\mathbf{B}\varphi_2^{(j)}$ , so  $\varphi_1^{(j)}-\mathbf{B}\varphi_2^{(j)}=\varphi_1^{(j)}-\varphi_1^{(j)}$ . Similarly, take the first  $n_1$  elements of  $\mathbf{v}^{(j)}$  ( $j=1,2,\dots,m$ ) to assign to  $\varphi_1^{(j)}$ , and set  $\varphi_0=\mathbf{0}$ , solving the large scale domain, then generate  $\varphi_2^{(j)}$ , and  $\varphi_2^{(j)}-\mathbf{D}\varphi_1^{(j)}=\varphi_2^{(j)}-\varphi_2^{(j)}$ , finally we get the iterative item  $\mathbf{A}\mathbf{v}^{(j)}$  that contains  $\varphi_1^{(j)}-\varphi_1^{(j)}$  and  $\varphi_2^{(j)}-\varphi_2^{(j)}$ .
- 3) Based on the Arnoldi method,  $\mathbf{v}^{(j+1)}$  can be obtained by  $\mathbf{v}^{(j)}$  and  $\mathbf{A}\mathbf{v}^{(j)}$ . Repeat step 2), we get  $\mathbf{v}^{(1)},\dots,\mathbf{v}^{(i)},\dots,\mathbf{v}^{(m+1)}$ . According to the GMRES method, the approximate solution  $\mathbf{x}^{(m)}$  is formed. Compute the residue  $\mathbf{r}^{(m)}=\mathbf{b}-\mathbf{A}\mathbf{x}^{(m)}$  and exit if  $\|\mathbf{r}^{(m)}\|_2$  is small enough. Otherwise,  $\mathbf{x}^{(0)}=\mathbf{x}^{(m)}$ ,  $\mathbf{v}^{(1)}=\mathbf{r}^{(m)}/\|\mathbf{r}^{(m)}\|_2$ , continue with step 2).

#### IV. SPECIFIC PARAMETERS OF THE MODEL

This paper take the ground electrode lies in Gezhouba of Ge-Shang HVDC Transmission Project as the case for calculation. The ground electrode is single ring with diameter as 500m and cross section diameter is 32mm, buried under ground at the depth of 2.5m. The current injected into the earth from ground electrode is 1200 A. The cross section of filler materials is 80 cm×80 cm. The resistivity of each earth layer is shown in Table I.

TABLE I  
HORIZONTAL MULTILAYER SOILS MODEL OF GEZHOUBA

Earth layer number	Resistivity ( $\Omega\cdot\text{m}$ )	Layer thickness
1	28	4
2	70	15
3	38	90
4	90	50
5	15	40
6	45	2500
7	230	5000
8	950	10000
9	300	$\infty$

#### V. RESULTS AND CONCLUSION

##### A. Calculation Results

In view of the earth model is horizontally stratified, the distribution of geoelectric current field is symmetrical. As a result, we just take the earth area that positive electrode lies in, i.e. half model of whole HVDC geoelectric current field model needs to be built, specific dimension parameter for earth model is shown in Figure 1.

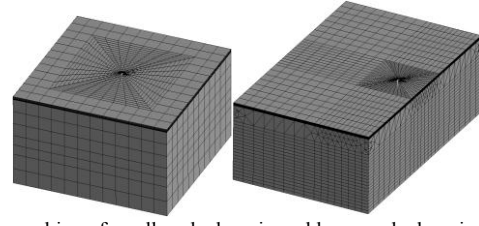


Fig. 2. The meshing of small scale domain and large scale domain

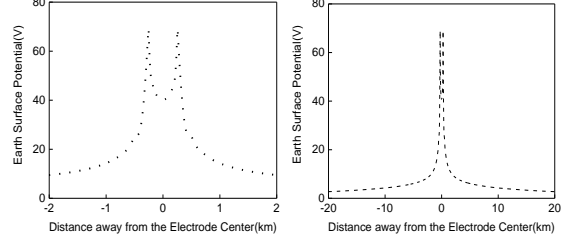


Fig. 3. Earth surface potential distributed in two paths

In addition to GMRES iterative algorithm, direct iteration and relaxation iteration are two commonly used methods. It only takes one time restart to obtain convergence. After the same number of solutions for each domain, the residue  $\mathbf{r}^{(m)}$  of direct iteration and the relaxation iteration are compared with the algorithm of the paper as shown in Table II.

TABLE II  
COMPARISON OF RESIDUE

Iterative approach	Residue $\mathbf{r}^{(m)}$
GMRES	6.347E-5
Direct iteration	1.353
0.1	28.31
0.2	22.81
0.3	18.13
0.4	14.11
0.5	10.70
0.6	7.870
0.7	5.570
0.8	3.753
0.9	2.365
Relaxation iteration	1
1.1	1.353
1.2	6.665
1.3	2.535
1.4	5.715E-2
1.5	2.422E-3
1.6	7.956E-5
1.7	2.654E-3
1.8	0.155 9
1.9	0.852 5
1.9	4.483

##### B. Conclusions

In this paper, DDM was used to separate the ground electrode from the whole model, and so that reduces the consumption of computer memory for solving whole model.

GMRES iterative algorithm for DDM iteration can be more efficient than direct iteration method and more feasible than relaxation iterative. This method provides a new solution for solving complex multi-scale problems.

#### REFERENCES

- [1] G. Aiello, S. Alfonzetti, and G. Borzi, "Efficient solution of skin-effect problems by means of the GMRES-accelerated FEM-BEM method," *IEEE. Trans. Magn.*, 2008, vol. 44, pp. 1274-1277.
- [2] A. Saitoh, A. Kamitani, "GMRES with new preconditioning for solving BEM-type linear system," *IEEE. Trans. Magn.*, 2004, vol. 40, pp. 1084-1087.